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ANALYTICAL STUDIES OF PHASE ESTIMATION TECHNIQUES

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Phase retrieval algorithms are presented for extended objects and for multiple images arising, for example, from observations at several wavelengths or several focal planes. The first experimental demonstration of phase retrieval is reported.

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1 INTRODUCTION

This technical report summarizes the work performed under contract No. F30602-77-C-0176 from January 15, 1979 to August 16, 1979. We report progress in the following areas:

Draper Work

Algorithm Development

1. DC measurement problem
2. Two focal planes or two wavelengths for extended objects
3. Smoothing techniques

Hughes Work

1. Noise
2. Blind Test

Experimental Work

One-D Results

2 DRAPER WORK

Personnel from Draper gave us a phase aberration that is a typical output from a simulation of the ITEK imaging model. We fit this aberration with a 14-term Zernicke polynomial expansion and constructed a point spread function (PSF) from the polynomial phase.

When we performed phase retrieval on the PSF at $\lambda = 8$ microns the phase was retrieved almost exactly. At $\lambda = 4$ microns, where the effective phase aberration is twice as large as at 8 microns, the algorithm did not perform as well. The residual phase produced an improved PSF but it was far from diffraction limited.

At an August 16 meeting of Draper and EIKONIX personnel, we decided to perform another round of phase retrieval on the improved PSF.

Details of this work appear in TM-9 and a report of the successful recovery of the phase in the above-mentioned second iteration will appear in the final report.

3 ALGORITHM DEVELOPMENT

3.1 DC Measurement Problem

Quantization of the observed PSF causes an error in the DC ($f = 0$) term of the optical transfer function (OTF). In simulations this was not a problem because the data was generated in floating-point form. However, the phenomenon appears in measurements from our experiment where the data is recorded only to 8 significant bits.

The current algorithm fits an observed OTF with an hypothesized OTF, both normalized to one at $f = 0$. Thus an error in the normalization can cause an incorrect, hypothesized OTF to fit the observed OTF better than the correct OTF.

The remedy is simple. We find the constant that scales the hypothesized OTF to best fit the observed OTF (in a mean square error sense). Calculation of the constant is straightforward. Details appeared in the May 1979 monthly report and will be updated for the final report.

3.2 Two Focal Planes or Two Wavelengths

This algorithm is particularly useful for extended objects. If the object's spectrum $O(f)$ is known, then the problem is nearly identical to that of the point object. The algorithm uses $O(f)$ and $O(f)P_c(f)$ in the block diagram of Figure 1. $P(f)$ and $P_c(f)$ are the observed and hypothesized OTF's.

If the object (scene) is unknown, then it must first be estimated based on additional information. Such information might come from an additional observation of the scene at another wavelength or at another focal position. Call the two observations $z_1(x)$ and $z_2(x)$ with Fourier transforms $Z_1(f)$ and $Z_2(f)$. Then an appropriate estimate of the object spectrum is

$$\hat{O}(f) = \frac{P_{1c}^*(f)Z_1(f) + P_{2c}^*(f)Z_2(f)}{|P_{1c}(f)|^2 + |P_{2c}(f)|^2} ,$$

where $P_{1c}(f)$ and $P_{2c}(f)$ are estimates of the system OTF's based on an estimate of the unknown phase aberration. This estimate is used in a slight modification to the algorithm of Figure 1 to find the best phase estimate.

This algorithm was tested, successfully, with a simulated, Gaussian object and noiseless observations at two focal positions. Testing with more general objects will be reported in the final report.

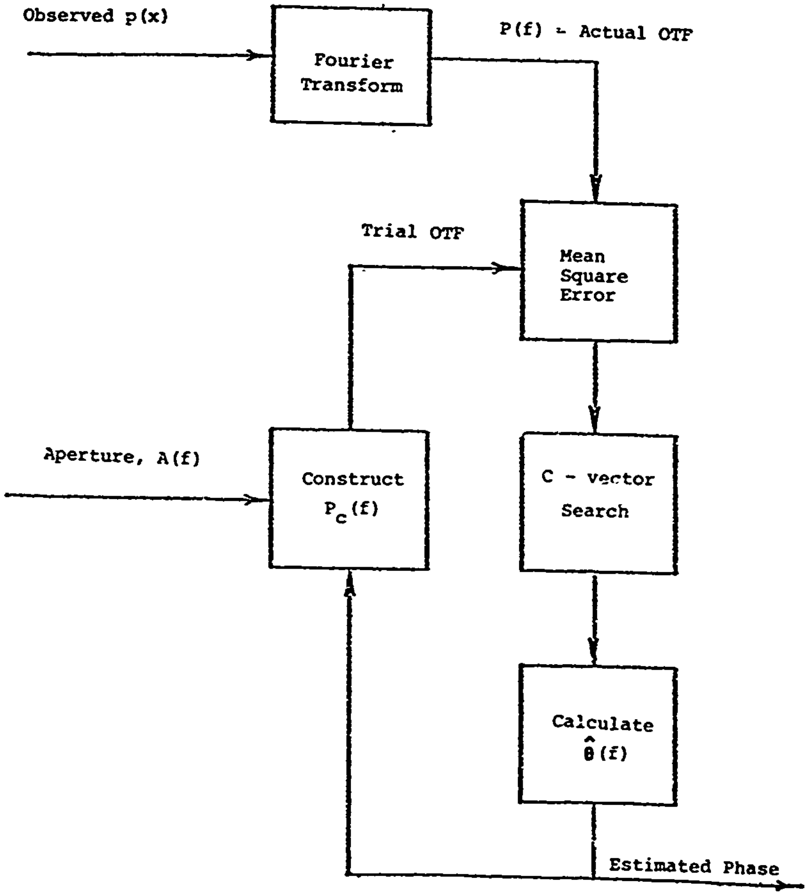

Figure 1 Parameter Search Algorithm

3.3 Smoothing Techniques

The error metric of Figure 1 has many local minima in the N-dimensional space of the Zernicke polynomial coefficients. In one-D notation the metric is

$$E(C) = \int |P(f) - P_C(f)|^2 \, df,$$

where P(f) is the observed OTF and $P_C(f)$ is the hypothesized OTF with parameter vector C. In earlier work under this contract (TM-9), we developed a method to smooth the metric by averaging over C-intervals. Thus, the averaged metric is

$$\overline{E(C)} = \int |P(f)|^2 df - 2 \, \mathrm{Re}\{\int P^*(f) \overline{P_C(f)} df\} + \int \overline{|P_C(f)|^2} df,$$

where the overbar denotes the C-average. In that earlier work, we showed plots of the second term for a particular, observed OTF, P(f), and for a one-dimensional variation of the C-vector.

In this reporting period we studied the third term to see how rapidly it varies with the several C-parameters. TM-4 is a report of this study. We found that the Zernicke polynomials for coma, 0 and 45 degree astigmatism and X and Y coma were relatively smooth for coefficients from about .2 to .6 (waves). This provided us with a rationale for selecting the interval (.25 to .5) for the grid search of the basic algorithm.

The result above is a by-product of the search for a good smoothing technique. We have neither programmed nor tested $\overline{E}(C)$, itself, to see if it provides a better metric -- one that can be searched more efficiently because it has fewer minima. This task has not been undertaken, as yet, because of its lower priority among other tasks to be accomplished.

4 WORK WITH HUGHES AIRCRAFT COMPANY

During this period we used phase retrieval algorithms developed under this contract in two separate tasks for Hughes Aircraft Company. The first was a study of the algorithm's performance with additive noise in the measured PSF and the second was a blind test. At the request of the contract monitor and with Hughes' permission, we briefed the government on this work on August 16, 1979.

In the first task we estimated the residual phase error vs. measurement noise in the PSF. We found that when the PSF is corrupted by signal-dependent noise with variance kp(x), it causes the residual phase (after correction) to increase gradually up to some value of k and then it increases rapidly. Figure 2 shows a typical plot of the residual phase's rms value vs. input noise level when the original phase has an rms value of 0.3 waves. This plot was obtained by Monte Carlo simulation. For k higher than 0.4 the residual phase increases dramatically.

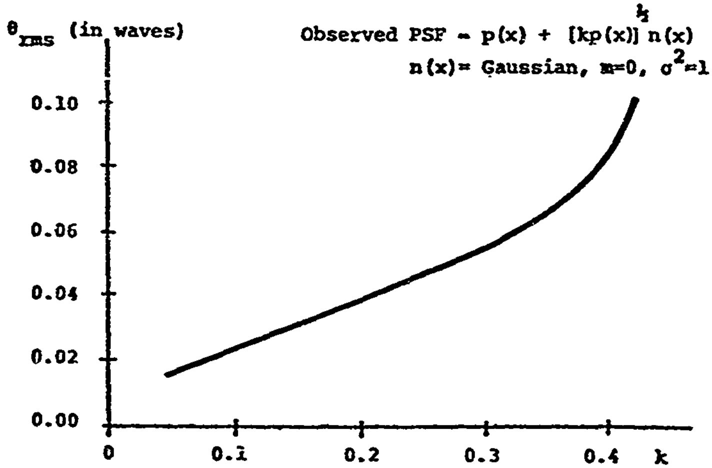

Figure 2 Residual θ_{rms} vs. measurement noise when input θ_{rms} is 0.3 waves.

The second task was a blind test where Hughes supplied us with a PSF from a system with unknown (to EIKONIX) phase aberration. We estimated the phase from the PSF. The results are summarized in Figure 3. The real phase (at the bottom) was given to EIKONIX by Hughes after the task was completed.

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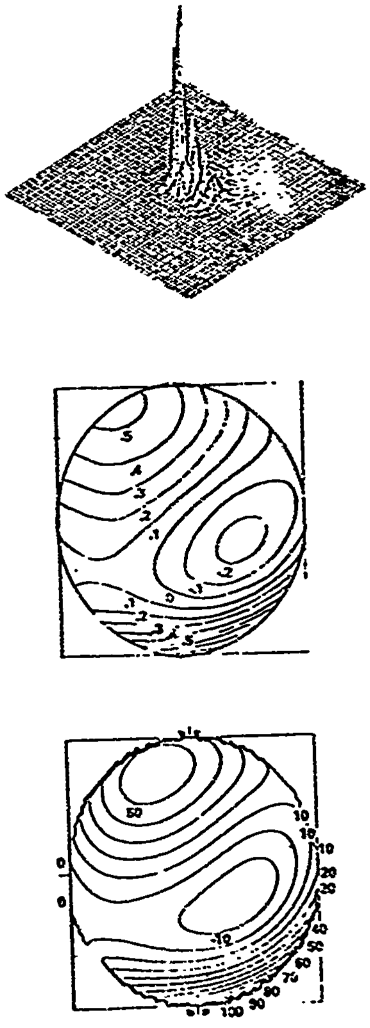

Measured PSF

Estimated Phase
1.0 = one wave

Actual Phase
100 = one wave

Figure 3 The Hughes Blind Test

5 EXPERIMENTAL WORK

The phase retrieval algorithm has been extensively tested in computer simulations, in which a point spread function from an aberrated optical system is generated by the computer and the PSF is passed to the phase retrieval program. Although this kind of work is a necessary first step, an experimental verification, in which an optical system with measured aberration is used to create the PSF, is needed to really prove the concept. An experiment was designed to provide that proof. We fabricated a phase aberration, measured it interferometrically (Figure 4), put it in an optical system to generate the PSF (Figure 5), submitted this PSF to the phase retrieval algorithm, and compared the resulting estimated phase (Figure 6) to the measured phase. The agreement is excellent and provides the first actual demonstration of phase retrieval in wavefront sensing.

A detailed presentation of the experiment will be given in the final report.

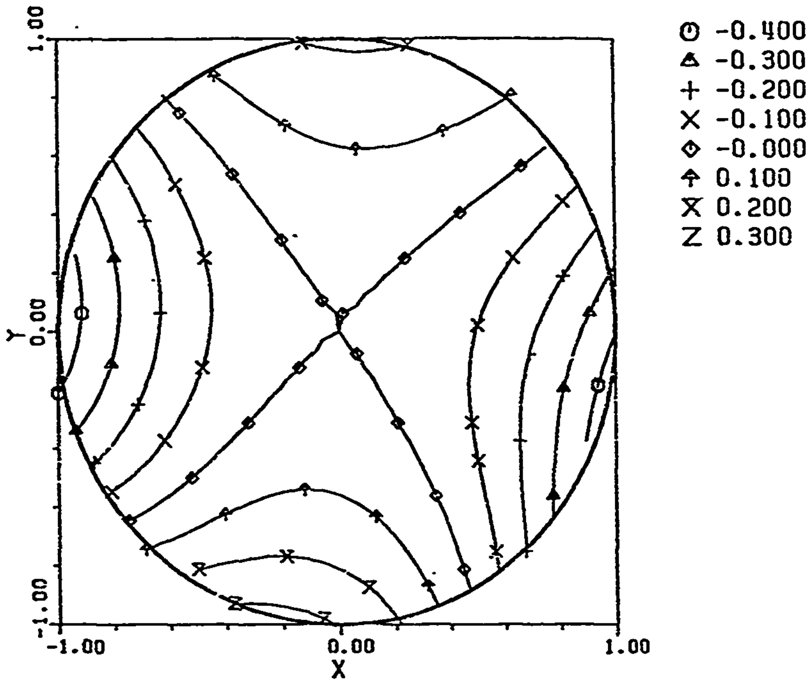

Figure 4 Interferometrically measured phase aberration reconstructed with Zernicke polynomials

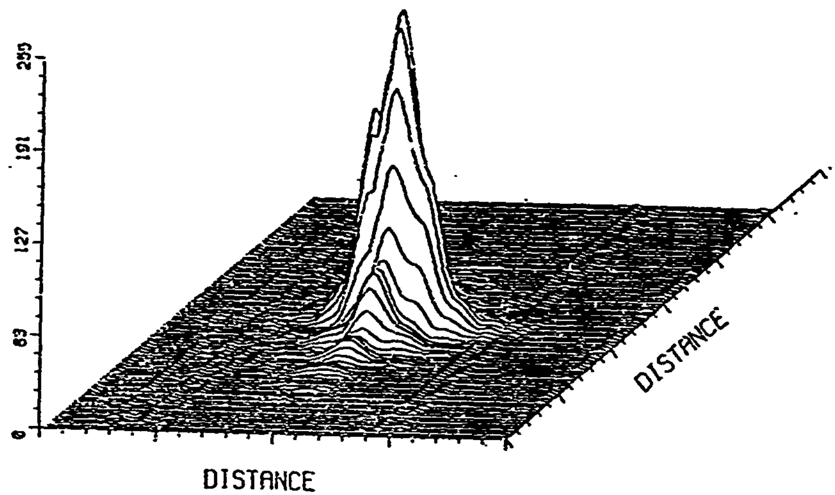

Figure 5 Aberrated PSF measured experimentally

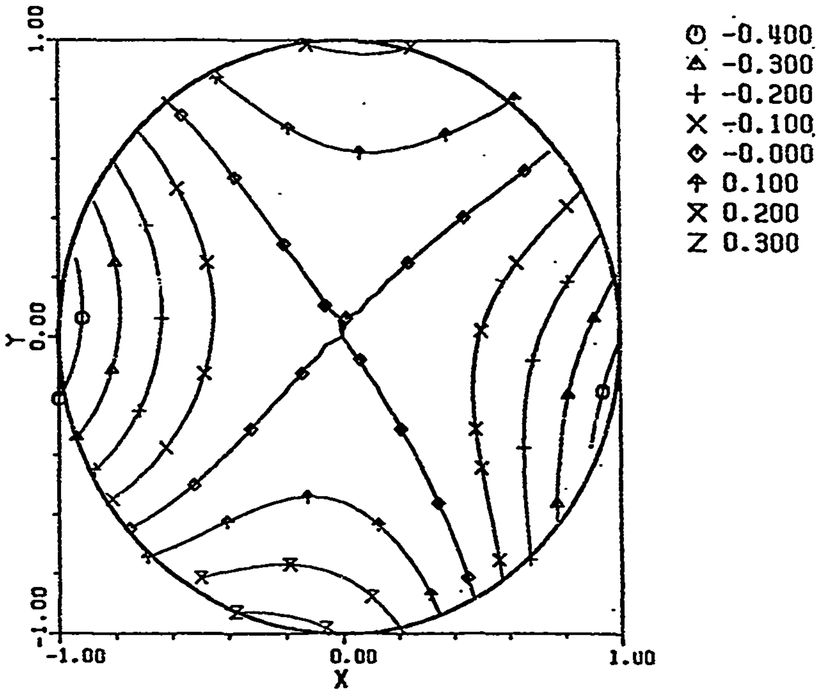

Figure 6 Estimated phase aberration corresponding to Figure 4.

6 ONE-D RESULTS

Two algorithms were developed for the one-dimensional phase retrieval problem. The first may be useful for small phase aberrations and the second appears to be a satisfactory solution for the noiseless, one-D problem.

Consider a small phase $\theta(f)$ for which the coherent system function $H(f)$ has the approximation

$$H(f) = e^{i\theta(f)} \quad , |f| < 1.$$

$$\approx 1 + i\theta(f), |f| < 1.$$

The system OTF is the autocorrelation of $H(f)$.

More generally, we define

$$C(f) = 1 + i\theta(f)$$

where $\theta(f)$ is real but not necessarily small. Its autocorrelation $P(f)$ is observed and the problem of phase retrieval is to find $\theta(f)$

We have shown that this problem has an exact solution and have developed an algorithm to find it. The solution is

$$\theta_K = A_{N-K+1} - \sum_{J=2}^{K-1} \theta_J \theta_{N+J-K} / \theta_N$$

$$\theta_{N-K+1} = B_{N-K+1} - \sum_{J=2}^{K} (\theta_{N+J-K} - \theta_J),$$

where the A_K and B_K are samples of the real and imaginary parts of the measured OTF (less the diffraction-limited OTF), and θ_K is a sample of $\theta(f)$.

A derivation of the result and several examples will appear in the final report.

The second algorithm attacks the more general problem where the sampled OTF is

$$P_K = \sum_{J=1}^{N-K+1} H_J^* H_{J+K-1} .$$

We can show that

$$\theta_K = \theta_N - F_K \pm G_K$$

and

$$\theta_{N-K+1} = B_K \pm G_K,$$

where

$$F_K = \text{angle of } D_K$$

$$G_K = \cos^{-1} (D_K/2),$$

and

$$D_K = P_{N-K+1} - \sum_{J=2}^{K-1} H_J^* H_{J+N-K}.$$

The choice of signs for θ_K yields a 2-fold ambiguity for each value of K or a 2^N-fold ambiguity for N θ_K's. The ambiguity is resolved by monitoring the magnitude of D_K. If, at any K, D_K has magnitude greater than 2, an incorrect sign has been made prior to the current value of K. This prunes the binary tree in such a way that the number of branches grows approximately linearly with K, an entirely manageable search. This is an observation, not a theorem, based on some searches of size N = 10, 20 and 64.

Details and examples will be given in the final report.

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